

# A Gravity Dual of Localized Tachyon Condensation in Intersecting Branes

Kazuyoshi Takahashi \*

*Yukawa Institute for Theoretical Physics, Kyoto University,  
Kyoto 606-8502, Japan*

## Abstract

The method of probe brane is the powerful one to obtain the effective action living on the probe brane from supergravity. We apply this method to the unstable brane systems, and understand the tachyon condensation in the context of the open/closed duality. First, we probe the parallel coincident branes by the anti-brane. In this case, the mass squared of the string stretched between the probe brane and one of the coincident branes becomes negative infinite in the decoupling limit. So that the dual open string field theory is difficult to understand. Next, we probe parallel coincident branes by a brane intersecting with an angle. In this case, the stretched strings have the tachyonic modes localized near the intersecting point, and by taking the appropriate limit for the intersection angle, we can leave mass squared of this modes negative finite in the decoupling limit. Then we can obtain the information about the localized tachyon condensation from the probe brane action obtained using supergravity.

---

\*E-mail: kazuyosi@yukawa.kyoto-u.ac.jp

# 1 Introduction

The importance of unstable brane systems has been recognized for long times [1]. One of the most successful utilization of the unstable brane systems was the tachyon condensation [2], which have given the many informations to understand the time-dependent dynamics of the decay of unstable brane system, and the tachyon condensation has been investigated using many powerful methods, for example, string field theory, effective field theory, the boundary CFT (or rolling tachyon), K-theory, and so on.

The open/closed duality also has been well known as the powerful method and concept in string theory. And so it is hoped that the open/closed duality is applied to the unstable brane system involving tachyon condensation. In this paper, we try to analyze some unstable branes and tachyon condensation using the probe brane method which is conjectured from the open/closed duality.

The open/closed duality is based on the following ideas. It is well known that the D-branes admit a perturbative description in terms of the open strings which are attached to them. On the other hand, the D-branes is thought to be a soliton solution of supergravity (or closed string theory). Using these two different descriptions, we can obtain the information of field theory on branes from the supergravity. The AdS/CFT correspondence is the most successful example [3, 4].

The method of probe brane is the important one based on the open/closed duality [5, 3, 6] \*. We consider a probe brane which moves close to a large number of coincident branes. Then we can obtain the effective action living on the probe brane using supergravity. This method has been confirmed to be valid for some examples not limited to the extremal cases. In this paper, we show that the probe brane method is useful in understanding the unstable brane systems and tachyon condensation.

At first, we probe the brane solution by the anti-brane. In this case, we show that the mass squared of the stretched strings become negative infinite in the decoupling limit ( $\alpha' \rightarrow 0$ ), so that the dual open string field theory is not well defined.

Next, we probe parallel coincident branes by a brane intersecting with an angle. The energy of the open strings stretched between probe brane and the one of the rest

---

\*The original references are contained within [5].

coincident branes are minimized by being confined to the intersection point, and these stretched strings involve tachyon modes localized near the intersecting point when the probe brane is close enough to the coincident branes. Interestingly, in the contrast to the case of the anti-brane probing, the mass squared of the localized tachyons leaves negative finite in the decoupling limit by taking an appropriate limit for the intersection angle. Then the effective action obtained from supergravity is nontrivial and involves the information about localized tachyon condensation. We analyze the localized tachyon condensation from the supergravity.

In the intersecting branes at angles, it is known that the recombination of branes occurs [7, 8, 9] \*, and this is interpreted as (local) tachyon condensation recently [9]. Our results from probe brane is not incompatible with the recombination process, and this also will be discussed.

There are the another interesting approaches toward the open/closed duality in unstable brane systems recently. For example, the open/closed duality at tree level in the rolling tachyon process [11] which insists that the tachyon matter in open string picture can be identified as the collective closed strings [12, 13, 14], the observation that in the matrix model of the non-critical string theory the matrix eigenvalue can be identified as the open string tachyon on an unstable  $D0$ -brane [15, 16, 17, 18], Schwarzschild black brane as the unstable brane systems [19, 20, 21, 22, 23], imaginary branes as the closed string states [24], time dependent holography in S-branes [25], the description of D-branes in the closed string field theory [26, 27], and so on. We believe that the method of the probe brane together with these works will give the some clues to the open/closed duality in unstable brane system.

The paper is organized as follows. In section 2, we review the method of probe brane and open/closed duality (gauge/gravity correspondence) briefly. Next, we probe the parallel coincident branes by the anti-brane and discuss about the dual field theory. In section 3, we probe parallel coincident branes by a brane intersecting with an angle and analyze the localized tachyon condensation from the effective action obtained using

---

\*The recombination of branes is well known to be an intriguing realization of Higgs phenomena in Standard model and so this process is also interesting phenomenologically. For example See [10] and their reference for recent progress.

supergravity. The relation with the recombination also is discussed in this section. The section 4 is devoted to conclusion.

Finally we comment on the interpretation of the probe brane action obtained in this paper briefly. In the anti-brane and intersecting brane probing, the stretched strings include the tachyonic modes. So that the energy scale in the action of the probe brane is not low compared to the masses of the all stretched strings. Therefore the action of the probe brane can not be regarded as the effective action obtained by integrating out the all fields whose energy scale is low compared to the one of the stretched strings, as is usually recognized in the probe brane [3]. Alternatively, the probe brane action should be regarded as the action which is obtained by integrating out only the fields generated by the stretched string. As we will see in section 3, the energy scale of the fields on the probe brane is proposed to be equal to the mass scale of the tachyon field around the true vacuum of the tachyon potential.

## 2 Brane and Anti-Brane Probing

The probing brane method is powerful one for investigating the dynamics of the large  $N$  gauge theory and this gave the important clues for the AdS/CFT correspondence [3, 4]. In section 2.1, we review the probing brane method and gauge/gravity correspondence briefly following [5, 3, 6]. In section 2.2, we replace probe brane with anti-brane and discuss about the dual open string field theory and point out the some problems. In this paper, we consider  $D3$ -brane mainly.

### 2.1 Brane Probe and Gauge/Gravity Correspondence

In this section, we review the method of probe brane following [5, 3, 6]. We consider  $N + 1$   $Dp$ -branes,  $N$  of which sit at the origin ( $r = 0$ ) and the last is the probe brane which sits at a distance  $r$ . The world-volume action at low energies is  $p + 1$ -dimensional  $U(N + 1)$  Yang-Mills theory broken down to  $U(N) \times U(1)$  by the expectation value of an adjoint scalar which represents the distance between the probe and the rest of branes. The strings stretched between the probe brane and one of the rest  $N$  branes are massive, with a mass  $m = r/(2\pi\alpha')$ .

We want to ignore the excited string states, which is performed by letting  $\alpha' \rightarrow 0$ . In order for the energies of stretched strings to remain finite in this limit, we need to approach the probe brane to origin in such a way that  $\hat{X}^I \equiv \frac{X^I}{\alpha'} = \text{fixed}$ , where  $X^I$  ( $I = p+1, \dots, 9$ ) is the transverse position of the probe  $Dp$ -brane. And, at same time, we need to take gauge coupling constant finite. These limits for  $Dp$ -brane are well known in the context of AdS/CFT and given in summary by

$$\begin{aligned}\alpha' &\rightarrow 0 \\ g_{YM}^2 &= 2(2\pi)^{p-2} g_s \alpha'^{\frac{p-3}{2}} = \text{fixed} ,\end{aligned}\tag{2.1}$$

and near-horizon limit

$$\hat{X}^I \equiv \frac{X^I}{\alpha'} = \text{fixed} ,\tag{2.2}$$

where  $g_s$  is string coupling.

On the other hand, D-branes are well described as a classical solution of the low-energy supergravity equations of motion in the region

$$\begin{aligned}g_s &\rightarrow 0, \quad N \rightarrow \infty \\ g_s N &= \text{fixed} \gg 1,\end{aligned}\tag{2.3}$$

We consider the  $D3$ -brane case mainly. In the region (2.3), the  $N$   $D3$ -branes is well described by the corresponding supergravity solution \*as

$$\begin{aligned}ds^2 &= H^{-1/2} \eta_{\alpha\beta} dx^\alpha dx^\beta + H^{1/2} \delta_{ij} dx^i dx^j, \\ e^\Phi &= 1, \\ F_5 &= dH^{-1} \wedge dx^0 \wedge \dots \wedge dx^3 + (dH^{-1} dx^0 \wedge \dots \wedge dx^3)^*,\end{aligned}\tag{2.4}$$

where  $\alpha, \beta = 0, \dots, 3$ ,  $i, j = 4 \dots 9$  and  $F_5$  is R-R five-form field strength. The warp factor  $H$  is given by

$$H(r) = 1 + \frac{4\pi g_s N \alpha'^2}{r^4} .\tag{2.5}$$

In the decoupling limit (2.1) and (2.2),

$$H(\hat{r}) = \frac{1}{(\alpha')^2} \frac{4\pi g_s N}{\hat{r}^4}\tag{2.6}$$

---

\*See the review [28] for example.

where  $\hat{r}^2 = (\hat{X}^I)^2$ , and we used the  $\hat{X}^I$ , which is finite in the limit (2.1), instead of the  $X^I$ .

In the parameter region (2.3), the  $N$   $D3$ -branes at origin ( $r = 0$ ) is represented by the brane solution (2.4), and so the probe  $D3$ -brane takes the interaction from curved background (2.4). In the gauge field side, this interaction comes from integrating out the massive fields generated by the open string which is stretched between probe brane and one of the rest branes. Thus, after integrating out these massive fields, the world-volume action on the probe brane should be equal to the Born-Infeld action in curved background (2.4). This suggestion is known as the open/closed duality (gauge/gravity correspondence) and has played an important roles also in AdS/CFT correspondence [3, 4].

The action of a probe  $D3$ -brane in a general supergravity background is given, for turning off the gauge field and Kalb-Ramond field, by

$$S = -\frac{1}{g_s(2\pi)^3\alpha'^2} \int d^4x \{e^{-\Phi} \sqrt{-\det \mathbf{P}(G_{\mu\nu})} + \mathbf{P}(C_4)\} \quad (2.7)$$

where  $\mathbf{P}$  denotes pull-back to the world-volume of bulk fields and  $C_4$  is the R-R 4-form potential. In the  $N$   $D3$ -branes background, we obtain the action on probe brane at leading order of the kinetic terms as

$$S = -\frac{1}{4\pi^2 g_{YM}^2} \int dx^4 \partial_\mu \hat{X}^I \partial^\mu \hat{X}^I, \quad (2.8)$$

where  $\mu = 0, \dots, 3$  is the world-volume direction.

It is confirmed in some examples not limited to extremal cases that results obtained from supergravity match with the results which is obtained by integrating out the stretched strings directly in the field theory side <sup>\*</sup>.

## 2.2 Anti-Brane Probing Brane Solution

In this section, we consider  $N$   $D3$ -branes and one  $\overline{D3}$ -brane, where  $N$   $D3$ -branes sit at the origin ( $r = 0$ ) and the  $\overline{D3}$ -brane sits at a distance  $r$  as a probe brane <sup>†</sup>. We investigate this system using the method of probe brane.

---

<sup>\*</sup>For example, see the reference in [5]

<sup>†</sup>This system was discussed originally in [29], but the relation with open/closed duality was not investigated there.

First, we consider the anti-brane probe from the supergravity. Because the R-R charge of the  $\overline{D3}$ -brane is opposite to the one of the  $D3$ -brane, the world-volume action on the  $\overline{D3}$ -brane probe in the background (2.4) is given by reversing the sign of Wess-Zumino term from the case of brane probe in section 2.1. Therefore the action on the anti-brane probe is given at leading order of the kinetic term by

$$S = -\frac{1}{4\pi^2 g_{YM}^2} \int d^4x \left( \partial_\mu \hat{X}^I \partial^\mu \hat{X}^I + \frac{4\hat{r}^4}{g_{YM}^2 N} \right). \quad (2.9)$$

As we have reviewed in section 2.1, it is expected that the probe action (2.9) is given by integrating out the fields generated by the open string stretched between  $\overline{D3}$  and one of the  $N$   $D3$ -branes. The only point we should notice is that the GSO projection for the  $3-\overline{3}$  strings<sup>\*</sup> is opposite to the one we chose for the  $3-3$  strings [1]. The mass of the lowest excitation mode of the  $3-\overline{3}$  strings is given by

$$m^2 = -\frac{1}{2\alpha'} + \left( \frac{\hat{r}}{2\pi} \right)^2. \quad (2.10)$$

Interestingly, the lowest mode of  $3-\overline{3}$  strings is tachyonic and it is expected that the anti-brane probe action (2.9) is the effective action on  $\overline{D3}$ -brane after integrating out these tachyon fields. However this anticipation is naive. We need to impose the decoupling limit (2.1) and (2.2) to ignore excited string states. In this limit, the mass squared (2.10) becomes negative infinite, so that the behavior of these fields is not well defined. This implies that the dual open string picture in the  $\overline{D3}$ -brane probing is unclear or invalid even though the probe action from supergravity is well defined.

In next section, we propose the brane configuration where the fields integrated out include tachyonic modes and these mass squared leaves finite in the decoupling limit (2.1). This brane configuration is the intersecting branes at angles, which has been discussed often in the context of tachyon condensation recently [9, 30, 31, 32].

### 3 Intersecting Branes and Open/Closed Duality

In this section, we probe parallel coincident branes by a brane intersecting with an intersection angle. As noted in section 2.1, the probe brane action obtained from

---

<sup>\*</sup>We denote the open string stretched between  $\overline{D3}$  and one of the  $N$   $D3$ -branes as  $3-\overline{3}$  string, and the one stretched between  $D3$ -branes as  $3-3$  string.

supergravity should be equal to the effective action after integrating out the fields generated by the stretched string. In this case, the stretched string include tachyon modes localized near the intersecting point. Interestingly, by taking appropriate limit for the intersection angle, the mass squared of this tachyonic mode leaves finite in the decoupling limit (2.1) in contrast to the case of the anti-brane probing. Using the probe brane action obtained from supergravity, we can analyze the condensation of the localized tachyon.

In section 3.1 and 3.2, we analyze this system from field theory side (open string side) and supergravity side (closed string side) respectively. In section 3.3, we discuss about the localized tachyon condensation using the probe brane action obtained from supergravity. In section 3.4, we comment on the relation with the recombination process in the intersecting branes [7, 8, 9].

### 3.1 Intersecting Brane Probe : Field Theory Side

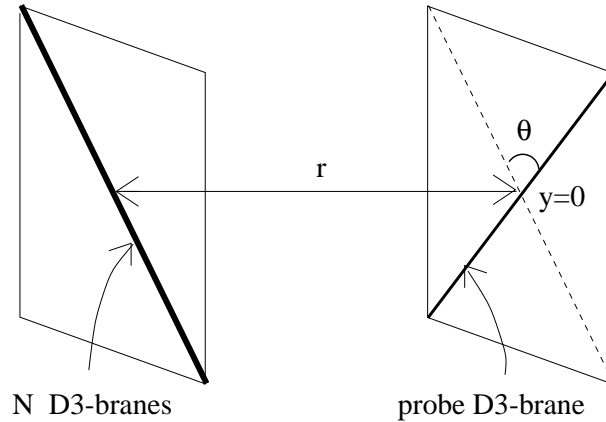


Figure 1: We probe  $N$  parallel  $D3$ -branes by a brane which has an intersection angle  $\theta$  in the  $(X^3, X^4)$ -plane, and we set the intersecting point to  $y \equiv X^3 = 0$ .

In this section, we probe  $N$  parallel coincident  $D3$ -branes by a brane which has an intersection angle  $\theta$ . The  $N$  parallel  $D3$ -branes are parameterized by world-volume coordinates  $X^0, X^1, X^2, X^3$ , and sit at the origin of the transverse spaces ( $X^4, \dots, X^9 = 0$ ). The probe brane is also parameterized by the  $X^0, X^1, X^2, X^3$  coordinates, but has



intersection angle  $\theta$  in the  $(X^3, X^4)$ -plane, so that  $X^4$  depends on  $X^3 \equiv y$  coordinate as \*

$$\begin{aligned} X^4 &= qy, \\ q &\equiv \tan \theta . \end{aligned} \quad (3.1)$$

The transverse distance between probe brane and the one of rest branes at the intersecting point ( $y = 0$ ) is denoted as  $r$ , or  $r \equiv \sqrt{\sum_{I=5}^9 (X^I)^2} \big|_{y=0}$ . See the figure 1 for this brane configuration. In this case, the energy of the open string stretched between probe  $D3$ -brane and the rest of  $N$   $D3$ -branes is minimized by being confined to the intersection point ( $y = 0$ ). It is known from world-sheet analysis [33, 34] that the mass squared of this stretched open string is given by

$$m^2 = \left(n - \frac{1}{2}\right) \frac{\theta}{\pi\alpha'} + \left(\frac{r}{2\pi\alpha'}\right)^2 \quad (3.2)$$

where  $n$  is a integer ( $n \geq 0$ )\*. Therefore, when  $r$  is small enough, the lowest mode of the stretched strings are tachyonic †.

In contrast to the case in section 2.2, in decoupling limit (2.1), the mass squared of the tachyon mode is not always negative infinite, and can be finite by taking the appropriate limit for  $\theta$  and  $X$  as

$$\begin{aligned} \hat{\theta} &\equiv \frac{\theta}{\alpha'} = \text{fixed} , \\ \hat{X}^I &\equiv \frac{X^I}{\alpha'} = \text{fixed} . \end{aligned} \quad (3.3)$$

where  $I = 5, \dots, 9$ . Then the mass squared (3.2) is given by

$$m^2 = \left(n - \frac{1}{2}\right) \frac{\hat{\theta}}{\pi} + \left(\frac{\hat{r}}{2\pi}\right)^2 , \quad (3.4)$$

where  $\hat{r} \equiv r/\alpha'$ . For  $\hat{r} = 0$ , the width of localized tachyons  $\delta y$  is given [8, 9] by ‡

$$\delta y \sim \sqrt{\hat{\theta}} . \quad (3.5)$$

---

\*We treat  $X^4$  as the fixed background. The fluctuation of  $X^4$  will be given later on.

\*Of course,  $\theta = 0$  case corresponds to the parallel D-branes case in section 2.1 and  $\theta = \pi$  case corresponds to the case in section 2.2, and they give the same mass relation with (3.2) respectively.

†See [35] for the related world-sheet analysis of intersecting branes.

‡This quantity has not been calculated for the case of  $\hat{r} \neq 0$ .

If we take another limit for the intersecting angle  $\theta$  as

$$\tilde{\theta} \equiv \frac{\theta}{(\alpha')^K} = \text{fixed} , \quad (3.6)$$

the  $\theta$  part does not contribute to the mass squared (3.2) for  $K > 1$ , and gives negative infinite mass to the mass squared for  $K < 1$ . These results are reflected to the probe action obtained from supergravity, as we will discuss in the next section.

As in section 2.1, it is anticipated from the open/closed duality that the Born-Infeld action on the probe brane in curved background (2.4) should be equal to the action on probe brane after integrating out the fields generated by the stretched strings. Interestingly, in this case, the lowest modes of the fields integrated out are tachyonic when the distance  $\hat{r}$  is small enough. Integrating out the tachyonic modes do not always imply that the effective action is not well-defined, as well known in the case of the brane-antibrane system [2] <sup>\*</sup>.

As we have denoted in the introduction, we can not regard the probe brane action as the effective action obtained by integrating out the all fields whose energy scale is low compared to the one of the stretched strings, as is usually recognized in the probe brane [3]. This occurs due to the tachyonic modes, whose mass is not defined, so that the energy scale in the action of the probe brane is not low compared to the masses of the all stretched strings. Therefore the action of the probe brane should be regarded as the action which is obtained by integrating out only the fields generated by the stretched string rather than the effective action. However the energy scale of the fields on the probe brane is unclear in this picture. We propose that this energy scale is equal to the mass scale of tachyon field around the true vacuum of the tachyon potential. This proposal is based on the ideas that the probe brane action is equivalent to the effective action which is defined at the true vacuum of the tachyon potential as we will see in section 3.3.

In the next section, we derive the probe brane action from supergravity and show how the limit (3.3) or (3.6) are reflected to the probe brane action.

---

<sup>\*</sup>In early days, some calculations to suggest that the tachyon condense and lead to the stable vacuum was performed [36].

### 3.2 Intersecting Brane Probe : Supergravity Side

We derive probe brane action from the supergravity as in section 2.1. The induced metric on the probe brane is given by

$$\begin{aligned} \mathbf{P}(G_{\mu\nu}) &= H^{-\frac{1}{2}} \left( \eta_{\mu\nu} + \frac{\partial X^I}{\partial x^\mu} \frac{\partial X^I}{\partial x^\nu} H \right) \text{ for } \mu, \nu \neq 3 \\ \mathbf{P}(G_{33}) &= H^{-\frac{1}{2}} \left\{ 1 + \left( q^2 + \left( \frac{\partial X^I}{\partial y} \right)^2 \right) H \right\} , \end{aligned} \quad (3.7)$$

where we denoted  $X^I$  as  $X^I = (X^5, \dots, X^9)$  and  $H$  is given by (2.5).

The action on the probe brane is given from (2.7) and (3.7) by

$$\begin{aligned} S &= -\frac{1}{g_s(2\pi)^3\alpha'^2} \int dx^4 H^{-1} \sqrt{\prod_{i=0}^3 \left( \eta_{ii} + \left( \frac{\partial X^I}{\partial x^i} \right)^2 H \right) \left\{ 1 + \left( q^2 + \left( \frac{\partial X^I}{\partial y} \right)^2 \right) H \right\}} \\ &+ \frac{1}{g_s(2\pi)^3\alpha'^2} \int dx^4 (H^{-1} - 1) . \end{aligned} \quad (3.8)$$

Here we ignored the contribution from the non-diagonal part of  $\mathbf{P}(G_{\mu\nu})$  in the determinant to consider only the leading order of the kinetic terms afterward.

First, we take the limit (2.1) and (3.3) in the action (3.8). In the limit (2.1) and (3.3),  $H$  is represented by

$$\begin{aligned} H &= 1 + \frac{4\pi g_s N \alpha'^2}{((qy)^2 + (X^I)^2)^2} \\ &= \frac{1}{\alpha'^2} \left( \alpha'^2 + \frac{g_{YM}^2 N}{((\hat{\theta}y)^2 + (\hat{X}^I)^2)^2} \right) \end{aligned} \quad (3.9)$$

where we used the relation  $q \equiv \tan\theta \rightarrow \theta$  in the limit (3.3). Therefore it is convenient to denote  $H$  as

$$H = \frac{\hat{H}}{\alpha'^2} , \quad (3.10)$$

where  $\hat{H}$  is given by

$$\begin{aligned} \hat{H} &\equiv \frac{g_{YM}^2 N}{\hat{r}^4} \\ \hat{r} &\equiv \sqrt{(\hat{\theta}y)^2 + (\hat{X}^I)^2} . \end{aligned} \quad (3.11)$$

We note that  $\hat{H}$  leaves finite in the limit (2.1) and (3.3). Using  $\hat{H}$  the probe action is given by

$$\begin{aligned}
S = & -\frac{1}{2\pi^2 g_{YM}^2} \int dx^4 \hat{H}^{-1} \sqrt{\prod_{i=0}^3 \left( \eta_{ii} + \left( \frac{\partial \hat{X}^I}{\partial x^i} \right)^2 \hat{H} \right) \left\{ 1 + \left( \hat{\theta}^2 + \left( \frac{\partial \hat{X}^I}{\partial y} \right)^2 \right) \hat{H} \right\}} \\
& + \frac{1}{2\pi^2 g_{YM}^2} \int dx^4 \hat{H}^{-1}
\end{aligned} \tag{3.12}$$

where we denoted as  $\hat{X}^I \equiv X^I/\alpha'$  and used  $g_{YM}$  in (2.1). This action is finite and well-defined in the limit (2.1) and (3.3), which has a physical implication in the contrast of the anti-brane case in section 2.2.

Now we consider only the leading order terms in the kinetic terms to analyze the dynamics of the probe brane. Then the probe brane action is given by

$$\begin{aligned}
S = & -\frac{1}{2\pi^2 g_{YM}^2} \int dx^4 \left\{ \frac{\sqrt{1 + \hat{\theta}^2 \hat{H}}}{2} \left( \frac{\partial \hat{X}^I}{\partial x^\alpha} \right)^2 + \frac{1}{2\sqrt{1 + \hat{\theta}^2 \hat{H}}} \left( \frac{\partial \hat{X}^I}{\partial y} \right)^2 \right\} \\
& - \frac{1}{2\pi^2 g_{YM}^2} \int dx^4 \left( \hat{H}^{-1} \sqrt{1 + \hat{\theta}^2 \hat{H}} - \hat{H}^{-1} \right)
\end{aligned} \tag{3.13}$$

where  $x^\alpha = (x^0, x^1, x^2)$ . Therefore the coefficients of kinetic terms depend on  $\hat{\theta}$  and  $\hat{H}$  in contrast with the case of the parallel branes (2.8).

For  $\hat{X}^I \gg 1$  and  $y \gg 1$ , because  $\hat{H} \simeq 0$  from (3.11), the action (3.13) is equal to the parallel brane case (2.8). On the other hand, for  $\hat{X}^I \ll 1$  and  $y \ll 1$  the equation (3.13) involves some physical implications, for example, localized tachyon condensation, which will be discussed in section 3.3.

We discuss about the another limit (3.6) for  $\theta$ . We see how the limit for the intersection angle is reflected to the probe brane action in supergravity. We represent the probe action (3.8) using  $\tilde{\theta}$  in (3.6) and  $\hat{X}$ . For  $K > 1$ , it is easy to show that the probe action is equal to the action of probe brane in the parallel  $D3$ -branes case in section 2.1. This is expected result from open string picture, because the mass of the fields integrated out is equal to the case in section 2.1 in decoupling limit.

The case of  $K < 1$  is somewhat nontrivial. In the field theory side, the mass of the field integrated out is negative infinite, so that the action on the probe brane is difficult to understand or invalid. Actually, the probe brane action obtained from

supergravity is compatible with the result from the open string picture. For  $K < 1$ , it is easy to confirm that the probe action (3.8) is order  $(\alpha')^{2(K-1)}$ , so that the action becomes infinite in the decoupling limit (2.1). Therefore this action is not well defined. This compatibility with open string picture is one of the evidence for the open/closed duality in the intersecting brane system.

Finally in this section, we consider the case where the fluctuation of  $X^4$  exists and be given by

$$X^4 = qy + \tilde{X}^4(x, y) , \quad (3.14)$$

where we denoted  $x$  as  $x = (X^0, X^1, X^2)$ . The part coming from this fluctuation in the probe brane action is given by

$$\tilde{S} = -\frac{1}{2\pi^2 g_{YM}^2} \int dx^4 \left\{ \frac{\sqrt{1 + \hat{\theta}^2 \hat{H}}}{2} \left( \frac{\partial \hat{X}^4}{\partial x^\alpha} \right)^2 + \frac{1}{2\sqrt{1 + \hat{\theta}^2 \hat{H}}} \left( 2\hat{\theta} \frac{\partial \hat{X}^4}{\partial y} + \left( \frac{\partial \hat{X}^4}{\partial y} \right)^2 \right) \right\} , \quad (3.15)$$

where we defined  $\hat{X}^4$  as  $\hat{X}^4 \equiv \tilde{X}^4/\alpha'$ . This term is different from the kinetic term of (3.13) due to the existence of the part of first derivative of  $y$ .

### 3.3 Localized Tachyon Condensation

In this section, we give a physical interpretation to the probe brane action (3.13). As we will discuss, this action involves the information about the (localized) tachyon condensation.

First, we remember the tachyon condensation in a brane-antibrane system [2, 1]. A system of coincident brane-antibrane has tachyonic modes generalized by the open string stretched between brane and anti-brane, and Sen conjectured that there is no degree of freedom of the open string around the true vacuum of tachyon potential  $V(T)$ . In this system, if we integrate out this open string tachyon, the partition function is dominated at the tachyon vacuum, that is

$$\int [DT] \exp(-S[T]) \sim \exp(-S[T_0]) \quad (3.16)$$

where  $T = T_0$  is the vacuum of tachyon potential. This implies that integrating out tachyon gets rid of the degrees of freedom of open strings.

Same ideas can be applied to the case of the intersecting branes. As we have denoted in section 3.1, for the probe brane being enough close to the origin, there are localized tachyon modes generated by the open strings stretched between probe brane and one of the rest branes. Of course, the potential of the localized tachyon is not bounded at tree level, but by integrating out the localized massive mode ( $n > 0$ ) in (3.4) this potential will be bounded. Then if we integrate out these tachyons localized at the intersecting point ( $y = 0$ ), we expect that the partition function dominates at the vacuum of the potential and that there are no degrees of freedom of open strings near the intersecting point ( $y = 0$ ) for  $\hat{X}^I = 0$ . These can be seen by the probe brane action obtained from supergravity.

As we noted in section 3.1, it is expected that the probe brane action (3.13) obtained from supergravity should be equal to the action on probe brane after integrating out the fields generated by the open string stretched between probe brane and one of the rest branes, which include tachyonic modes for probe brane enough close to origin. Therefore, in this case, the action (3.13) should not have degrees of freedom of open string near the intersecting point, namely  $\hat{X}^I \ll 1$  and  $y \ll 1$ .

Actually we can see the absence of the degrees of freedom of open strings in the probe brane action (3.13). At first, we consider about the potential part, which is given by

$$V_{\text{pot}} = \frac{1}{2\pi^2 g_{YM}^2} \left( \hat{H}^{-1} \sqrt{1 + \hat{\theta}^2 \hat{H}} - \hat{H}^{-1} \right). \quad (3.17)$$

Then, because  $V_{\text{pot}} \rightarrow \hat{\theta}/(2\pi^2 g_{YM}^2)$  for  $\hat{X}^I \rightarrow 0$  and  $y \rightarrow 0$ , the potential part is almost a constant in the region  $\hat{X}^I \ll 1$  and  $y \ll 1$ .

Because the potential part is almost constant, the kinetic terms (3.18) should involve the information about the dynamics of the probe brane. The part of kinetic terms in (3.13) is given by

$$S_{\text{kin}} = -\frac{1}{2\pi^2 g_{YM}^2} \int dx^4 \left\{ \frac{\sqrt{1 + \hat{\theta}^2 \hat{H}}}{2} \left( \frac{\partial \hat{X}^I}{\partial x^\alpha} \right)^2 + \frac{1}{2\sqrt{1 + \hat{\theta}^2 \hat{H}}} \left( \frac{\partial \hat{X}^I}{\partial y} \right)^2 \right\}, \quad (3.18)$$

where  $x^\alpha = (x^0, x^1, x^2)$ ,  $y = x^3$  and  $I = 5, \dots, 9$ . Because  $\hat{H} \gg 1$  in this case, the coefficient of  $\partial \hat{X}^I / \partial x^\alpha$  is large. This implies that the  $x^\alpha$  dependence of  $\hat{X}^I$  is

classical and does not have quantum fluctuations \*. Therefore for  $\hat{X}^I \ll 1$  and  $y \ll 1$ , the degrees of freedom of  $\hat{X}^I$  along the directions of  $x^\alpha$  have been lost, so that  $\hat{X}^I$  fluctuate following only  $y$ .

On the other hand, for  $\hat{X}^I \ll 1$  and  $y \ll 1$ , the coefficient of the  $\partial\hat{X}^I/\partial y$  becomes very small. This is similar to the absence of open string in the tachyon condensation of non-BPS brane.

To see this similarity, we remember the tachyon condensation in the non-BPS branes. In the case of the non-BPS  $p$ -branes \*, the world-volume action in flat space is given [37, 11] by

$$S = - \int d^{p+1}x V(T) \sqrt{-\det(\eta_{\mu\nu} + \partial_\mu X^i \partial_\nu X^i + \partial_\mu T \partial_\nu T + 2\pi\alpha' F_{\mu\nu})} , \quad (3.19)$$

where  $V(T)$  is tachyon potential,  $X^i$  ( $i = p+1, \dots, 9$ ) is the transverse scalar, and we have omitted the Wess-Zumino. Because  $V(T) \rightarrow 0$  for  $T \rightarrow T_0$ , the coefficient of the  $\partial X^I/\partial x^\mu$  approach zero in the process of tachyon condensation. And so the non-BPS brane action after the tachyon condensation has the similar behavior with the kinetic term  $\partial\hat{X}^I/\partial y$  in intersecting branes. This implies that the degrees of freedom of open string along the direction  $y$  in intersecting branes are absent in the sense similar with the case of the non-BPS brane †.

In summary, the probe brane action (3.18) seems to imply that the degrees of freedom of open string near the intersecting point become absent due to the localized tachyon condensation ‡. However, about how degrees of freedom become absent, the case of the  $x^\alpha$  dependence and one of the  $y$  dependence of  $\hat{X}^I$  differ from each other. This physical implication has been unclear yet.

Finally in this section, we point out the naive point in this subsection. The mass relation (3.4) is the one at tree level. Therefore, the tachyonic modes at tree level can become massless or massive due to the loop correction, so that the localized tachyon

---

\*This is similar to the case of zero temperature in thermal system.

\*The  $p$  is odd for Type IIA string theory and even for Type IIB string theory.

†In the case of non-BPS brane, it is proposed that the degrees of freedom of the open string don't die in tachyon condensation, but leave as the ones of the string fluid [38], which is interpreted as the collective closed strings recently [12, 13, 14].

‡The kinetic term of the fluctuation  $\hat{X}^4$  (3.15) is not similar to the non-BPS case due to the part of the first derivative of  $y$ . This result has not been understood physically.

condensation could not occur. But, as we noted before, it seems that the probe brane action (3.18) shows the absence of degrees of freedom of open strings. This argument is not rigorous one, but we believe that it is a evidence of the occurrence of the localized tachyon condensation <sup>§</sup>.

### 3.4 On Recombination of Intersecting Branes

In the intersecting branes at angles, it is known that the recombination of branes occurs [7]. The recombination can be analyzed by the effective field theory on the branes [8, 9], and it was interpreted as local tachyon condensation recently in [9]. See figure 2.

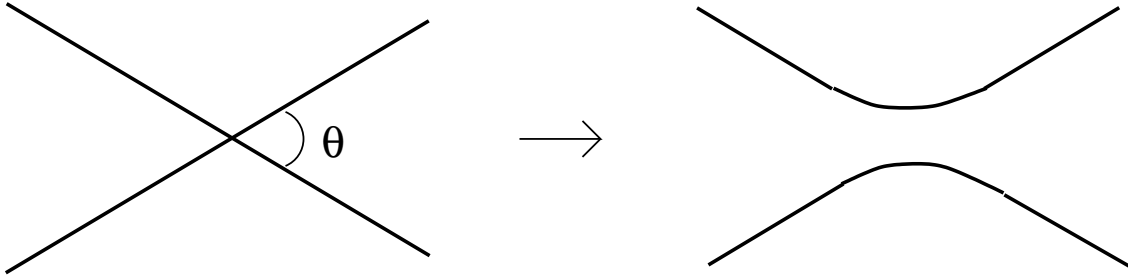


Figure 2: The intersecting branes are recombined due to the tachyon mode localized at the intersecting point.

In the same way, in our cases, where  $N$  branes are parallel to each other and only a probe brane is intersected at an angle, it is expected that the recombination of the probe brane with one of the rest branes will occur. However, the probe brane action does not show the recombination of branes. At first sight, this seems to imply the incompatibility between the recombination and our results from supergravity, but actually these two results are compatible.

To see this, we remember how the recombination can be understood using effective field theory, following [9]. At first, it is convenient that we consider the system consisting of two intersecting  $D3$ -branes which has an intersection angle  $\theta$  in  $(X^3, X^4)$ -plane,

---

<sup>§</sup>In the paper [39] the supergravity solutions dual to microstates of the D1-D3-D5 system with nonzero B field moduli is given. Because the angles of branes are T-dual to the B field, the relation with our papers and the paper [39] is interesting to investigate.



and both branes are parameterized by the coordinate  $x \equiv (X^0, X^1, X^2)$  and  $y \equiv X^3$ . The transverse scalar field is represented by

$$X^4 = \begin{pmatrix} qy & 0 \\ 0 & -qy \end{pmatrix} \quad (3.20)$$

where

$$q \equiv \tan(\theta/2) . \quad (3.21)$$

The non-diagonal parts of  $X^4$  exists as the fluctuation  $T(x, y)$  as

$$X^4 = \begin{pmatrix} qy & T(x, y) \\ T(x, y) & -qy \end{pmatrix} . \quad (3.22)$$

This fluctuation includes the tachyonic mode, and now we focus on this mode. The tachyonic mode rolls the tachyon potential, so that this mode is time dependent and given by

$$T(x, y) = C \exp\left(-\frac{qy^2}{2\pi\alpha'}\right) e^{\sqrt{\frac{q}{\pi\alpha'}} X^0} , \quad (3.23)$$

where  $C$  is a constant. The recombination is realized by diagonalizing  $X^4$ , and the diagonalized position is given by

$$X^4 = \begin{pmatrix} \sqrt{q^2 y^2 + T(x, y)^2} & 0 \\ 0 & -\sqrt{q^2 y^2 + T(x, y)^2} \end{pmatrix} . \quad (3.24)$$

Thus, in effective field theory, we does not integrate out the tachyon mode and this mode evolves following the equation of motion. On the other hand, in the probe brane method, the tachyonic mode needs to be integrated out, that is, the tachyon mode must sit at the vacuum of the tachyon potential. Therefore we haven't analyzed the time evolution of branes' positions in probe brane method. This implies that the subjects treated in effective field theory are not equal to the ones treated in probing brane method, so that these two results are compatible.

Then how the recombination is understood in probe brane method ? The diagonalization of the matrix  $X^4$  representing the branes' positions needs to be taken to realize the recombination. But this prescription is unclear in the probe brane method, because the non-diagonal part of the matrix  $X^4$  must be integrated out. And so, understanding recombination from supergravity is difficult and that is a future problem.

Finally, we emphasize that the subject of this paper is not to study the dynamics of the intersecting branes (recombinations and so on), but to understand and analyze the localized tachyon condensation in the intersecting branes in the context of open/closed duality even if we treat brane configuration as the fixed background.

## 4 Conclusions

We have applied the probe brane method to the unstable brane systems, in particular intersecting branes at angles. In this system, we can take an appropriate limit for the intersection angle with the decoupling limit  $\alpha' \rightarrow 0$  to leave the mass squared of the localized tachyons negative finite. So that we have obtained the probe brane action from supergravity which involves the information about localized tachyon condensation, though some unclear and naive problems have been left. We hope that a lot of techniques in AdS/CFT correspondence will be applied to the intersecting branes, or another unstable brane systems.

## Acknowledgments

I would like to thank my colleagues in Yukawa Institute for Theoretical Physics for discussions and encouragement. I am especially grateful to K. Hotta, H. Kajiura, S. Sugimoto, D. Tomino for useful discussions.

## References

- [1] A. Sen, “Non-BPS states and branes in string theory,” arXiv:hep-th/9904207.
- [2] A. Sen, “Stable non-BPS bound states of BPS D-branes,” JHEP **9808**, 010 (1998) [arXiv:hep-th/9805019].  
A. Sen, “Tachyon condensation on the brane antibrane system,” JHEP **9808**, 012 (1998) [arXiv:hep-th/9805170].

- [3] J. M. Maldacena, “The large N limit of superconformal field theories and supergravity,” *Adv. Theor. Math. Phys.* **2**, 231 (1998) [*Int. J. Theor. Phys.* **38**, 1113 (1999)] [arXiv:hep-th/9711200].
- [4] S. S. Gubser, I. R. Klebanov and A. M. Polyakov, “Gauge theory correlators from non-critical string theory,” *Phys. Lett. B* **428**, 105 (1998) [arXiv:hep-th/9802109].  
E. Witten, “Anti-de Sitter space and holography,” *Adv. Theor. Math. Phys.* **2**, 253 (1998) [arXiv:hep-th/9802150].
- [5] J. M. Maldacena, “Branes probing black holes,” *Nucl. Phys. Proc. Suppl.* **68**, 17 (1998) [arXiv:hep-th/9709099].
- [6] E. Imeroni, “The gauge/string correspondence towards realistic gauge theories,” arXiv:hep-th/0312070.
- [7] J. Polchinski, “String Theory II,” Cambridge University Press (1998)
- [8] A. V. Morosov, “Classical decay of a non-supersymmetric configuration of two D-branes,” *Phys. Lett. B* **433**, 291 (1998) [arXiv:hep-th/9803110].
- [9] K. Hashimoto and S. Nagaoka, “Recombination of intersecting D-branes by local tachyon condensation,” *JHEP* **0306**, 034 (2003) [arXiv:hep-th/0303204].
- [10] C. Kokorelis, “Exact standard model structures from intersecting D5-branes,” *Nucl. Phys. B* **677**, 115 (2004) [arXiv:hep-th/0207234].  
C. Kokorelis, “Exact standard model structures from intersecting branes,” arXiv:hep-th/0210004.
- [11] A. Sen, “Rolling tachyon,” *JHEP* **0204**, 048 (2002) [arXiv:hep-th/0203211].  
A. Sen, “Tachyon matter,” *JHEP* **0207**, 065 (2002) [arXiv:hep-th/0203265].  
A. Sen, “Field theory of tachyon matter,” *Mod. Phys. Lett. A* **17**, 1797 (2002) [arXiv:hep-th/0204143].
- [12] A. Sen, “Open and closed strings from unstable D-branes,” *Phys. Rev. D* **68**, 106003 (2003) [arXiv:hep-th/0305011].

- [13] A. Sen, “Open-closed duality at tree level,” *Phys. Rev. Lett.* **91**, 181601 (2003) [arXiv:hep-th/0306137].
- [14] H. U. Yee and P. Yi, “Open/closed duality, unstable D-branes, and coarse-grained closed strings,” arXiv:hep-th/0402027.
- [15] J. McGreevy and H. Verlinde, “Strings from tachyons: The  $c = 1$  matrix reloaded,” *JHEP* **0312**, 054 (2003) [arXiv:hep-th/0304224].
- [16] I. R. Klebanov, J. Maldacena and N. Seiberg, “D-brane decay in two-dimensional string theory,” *JHEP* **0307**, 045 (2003) [arXiv:hep-th/0305159].
- [17] M. R. Douglas, I. R. Klebanov, D. Kutasov, J. Maldacena, E. Martinec and N. Seiberg, “A new hat for the  $c = 1$  matrix model,” arXiv:hep-th/0307195.
- [18] T. Takayanagi and N. Toumbas, “A matrix model dual of type 0B string theory in two dimensions,” *JHEP* **0307**, 064 (2003) [arXiv:hep-th/0307083].
- [19] U. H. Danielsson, A. Guijosa and M. Kruczenski, “Brane-antibrane systems at finite temperature and the entropy of black JHEP **0109**, 011 (2001) [arXiv:hep-th/0106201].  
U. H. Danielsson, A. Guijosa and M. Kruczenski, “Black brane entropy from brane-antibrane systems,” *Rev. Mex. Fis.* **49S2**, 61 (2003) [arXiv:gr-qc/0204010].
- [20] A. Guijosa, H. H. Hernandez Hernandez and H. A. Morales Tecotl, “The entropy of the rotating charged black threebrane from a brane-antibrane system,” arXiv:hep-th/0402158.
- [21] O. Saremi and A. W. Peet, “Brane-antibrane systems and the thermal life of neutral black holes,” arXiv:hep-th/0403170.
- [22] O. Bergman and G. Lifschytz, “Schwarzschild black branes from unstable D-branes,” arXiv:hep-th/0403189.
- [23] S. Kalyana Rama, “A description of Schwarzschild black holes in terms of intersecting M-branes and antibranes,” arXiv:hep-th/0404026.

- [24] D. Gaiotto, N. Itzhaki and L. Rastelli, “Closed strings as imaginary D-branes,” arXiv:hep-th/0304192.
- [25] F. Quevedo, G. Tasinato and I. Zavala, “S-branes, negative tension branes and cosmology,” arXiv:hep-th/0211031.  
C. P. Burgess, P. Martineau, F. Quevedo, G. Tasinato and I. Zavala C., “Instabilities and particle production in S-brane geometries,” JHEP **0303**, 050 (2003) [arXiv:hep-th/0301122].  
A. Maloney, A. Strominger and X. Yin, “S-brane thermodynamics,” JHEP **0310**, 048 (2003) [arXiv:hep-th/0302146].
- [26] I. Kishimoto, Y. Matsuo and E. Watanabe, “Boundary states as exact solutions of (vacuum) closed string field theory,” Phys. Rev. D **68**, 126006 (2003) [arXiv:hep-th/0306189].  
I. Kishimoto, Y. Matsuo and E. Watanabe, “A universal nonlinear relation among boundary states in closed string field theory,” Prog. Theor. Phys. **111**, 433 (2004) [arXiv:hep-th/0312122].
- [27] T. Asakawa, S. Kobayashi and S. Matsuura, “Closed string field theory with dynamical D-brane,” JHEP **0310**, 023 (2003) [arXiv:hep-th/0309074].
- [28] A. W. Peet, “TASI lectures on black holes in string theory,” arXiv:hep-th/0008241.
- [29] C. P. Burgess, P. Martineau, F. Quevedo and R. Rabadan, “Branonium,” JHEP **0306**, 037 (2003) [arXiv:hep-th/0303170].
- [30] S. Nagaoka, “Fluctuation analysis of non-Abelian Born-Infeld action in the background intersecting D-branes,” Prog. Theor. Phys. **110**, 1219 (2004) [arXiv:hep-th/0307232].
- [31] K. Hashimoto and W. Taylor, “Strings between branes,” JHEP **0310**, 040 (2003) [arXiv:hep-th/0307297].

- [32] S. Nagaoka, “Higher dimensional recombination of intersecting D-branes,” JHEP **0402**, 063 (2004) [arXiv:hep-th/0312010].
- [33] M. Berkooz, M. R. Douglas and R. G. Leigh, “Branes intersecting at angles,” Nucl. Phys. B **480**, 265 (1996) [arXiv:hep-th/9606139].
- [34] H. Arfaei and M. M. Sheikh Jabbari, “Different D-brane interactions,” Phys. Lett. B **394**, 288 (1997) [arXiv:hep-th/9608167].  
M. M. Sheikh Jabbari, “Classification of different branes at angles,” Phys. Lett. B **420**, 279 (1998) [arXiv:hep-th/9710121].
- [35] N. Ohta and P. K. Townsend, “Supersymmetry of M-branes at angles,” Phys. Lett. B **418**, 77 (1998) [arXiv:hep-th/9710129]. T. Kitao, N. Ohta and J. G. Zhou, “Fermionic zero mode and string creation between D4-branes at angles,” Phys. Lett. B **428**, 68 (1998) [arXiv:hep-th/9801135].
- [36] K. Bardakci and M. B. Halpern, “Explicit Spontaneous Breakdown In A Dual Model,” Phys. Rev. D **10**, 4230 (1974).  
K. Bardakci and M. B. Halpern, “Explicit Spontaneous Breakdown In A Dual Model. 2. N Point Functions,” Nucl. Phys. B **96** (1975) 285.
- [37] A. Sen, “Supersymmetric world-volume action for non-BPS D-branes,” JHEP **9910**, 008 (1999) [arXiv:hep-th/9909062].  
M. R. Garousi, “Tachyon couplings on non-BPS D-branes and Dirac-Born-Infeld action,” Nucl. Phys. B **584**, 284 (2000) [arXiv:hep-th/0003122].  
E. A. Bergshoeff, M. de Roo, T. C. de Wit, E. Eyras and S. Panda, “T-duality and actions for non-BPS D-branes,” JHEP **0005**, 009 (2000) [arXiv:hep-th/0003221].  
J. Kluson, “Proposal for non-BPS D-brane action,” Phys. Rev. D **62**, 126003 (2000) [arXiv:hep-th/0004106].  
M. R. Garousi, “On-shell S-matrix and tachyonic effective actions,” Nucl. Phys. B **647**, 117 (2002) [arXiv:hep-th/0209068].  
D. Kutasov and V. Niarchos, “Tachyon effective actions in open string theory,” Nucl. Phys. B **666**, 56 (2003) [arXiv:hep-th/0304045].

- [38] G. W. Gibbons, K. Hori and P. Yi, “String fluid from unstable D-branes,” Nucl. Phys. B **596**, 136 (2001) [arXiv:hep-th/0009061].  
O. K. Kwon and P. Yi, “String fluid, tachyon matter, and domain walls,” JHEP **0309**, 003 (2003) [arXiv:hep-th/0305229].
- [39] O. Lunin, S. D. Mathur, I. Y. Park and A. Saxena, “Tachyon condensation and ‘bounce’ in the D1-D5 system,” Nucl. Phys. B **679**, 299 (2004) [arXiv:hep-th/0304007].